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VIT
Vellore Institute of Technology

Final Assessment Test (FAT) – June 2022

Programme	B.Tech	Semester	Winter Semester 2021-22
Course Title	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
Faculty Name	Prof. Revathi G K	Slot	C2+TC2+TCC2
		Class Nbr	CH2021222300582
Time	3 Hours	Max. Marks	100

Part A (10 X 10 Marks)
Answer any 10 questions

Co90 = 0

1. Solve $(D^2 + 2D + 1)y = \frac{e^{-x}}{x^2}$ by the method of variation of parameter. [10]
2. Solve $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$ [10]
3. (a) Find the Laplace transform of the saw tooth wave function of a period T defined as: $f(t) = \frac{100t}{T},\ for\ 0 < t < T$. (5 marks) [10]
   (b) Find the Laplace transform of $f(t) = e^t u(t-2) + t\delta(t-3)$ (5 marks)
4. Solve the following initial value problem by using Laplace transform: $2\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 2y = e^{-2t}$ with $y(0) = 1\ and\ y'(0) = 1$ [10]
5. Verify Parseval's identity for $f(x) = e^{-x}, x > 0$ in Fourier transform [10]
6. Using transform techniques, evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+25)(x^2+49)}$. [10]
7. Express the function $f(x) = \begin{Bmatrix} 0, & for\ -\pi < x < 0 \\ 1 & for\ 0 < x < \pi \end{Bmatrix}$ as Fourier series where $f(x + 2\pi) = f(x)$ .Hence find the value of $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$ [10]
8. Express the function $f(x) = x^2$ in $(0, a)$as a Fourier series and h ence find the value of the series $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$ [10]
9. (a) Find the Z-transform of $f(k) = \sin(\alpha k),\ k \geq 0$. (5 Marks) [10]
   (b) Find the inverse Z-transform of the function $\frac{z}{(z+3)^2(z-2)}, |z| > 3$ (5 Marks)
10. Solve the difference equations $y(k+2) - 5y(k+1) + 6y(k) = k$ by using using Z-transform with $y(0) = 0, y(1) = 0$ [10]
11. (a) Solve the following partial differential equations using Laplace transform $\frac{\partial y}{\partial t} + \frac{\partial y}{\partial x} + y = 0,\ for\ x > 0,\ t > 0\ with\ y(0,t) = \sin(t)\ and\ y(x,0) = 0$ (5 marks) [10]
    (b) Using convolution theorem find $f(t)$ if $L(f(t)) = \frac{1}{(s+1)(s^2+4)}$ (5 marks)
12. (a) Solve $(5 + 2x)^2 \frac{d^2y}{dx^2} - 6(5 + 2x)\frac{dy}{dx} + 8y = 0$ (5 Marks) [10]
    (b) Form the partial differential equation by eliminating f from $z^2 - 2xy = f(x^2 + y^2 + z^2)$ (5 marks)

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