Reg. No.: [redacted]

Name : [redacted]

VIT

Vellore Institute of Technology

Continuous Assessment Test -1: September 2022

Programme	: B.Tech.	Semester	: Fall 2022-23
Course Title	: Complex Variables and Linear Algebra	Code	: BMAT201L
Class No.	: CH2022231001185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197	Slot	: A2+TA2+TAA2
Faculty (s)	: Dr Jaganathan B, Dr Manivannan A, Dr. Felix A Dr Dhivya M, Dr Sudip Debnath, Dr Durga N Dr Prasanna Lakshmi M, Dr Harshavarthini, Dr Ashish Kumar, Dr Kamalesh, Dr Sushmitha, Dr Amit Kumar Rahul, Dr Balaji S	Max. Marks	: 50
		Time	: 90 Minutes

Answer ALL questions

Q.No.	Sub. Sec.	Question Description	Marks
1.	a)	Does the harmonic conjugate of the function $v(x,y) = \log_e((x-1)^2 + y^2)$ exist? Justify.	4
	b)	If $u(x,y) = e^{-2xy}\sin(x^2 - y^2)$ is the real part of an analytic function $f(z) = u + iv$, then find the imaginary part f. Also, determine $f'(z)$.	6
2.	a)	If $\phi(x,y) = x^2 - y^2 - 2xy - 2x - y - 1$ is the velocity potential of a incompressible fluid flow through a conduit, then calculate the complex potential $w = \phi(x,y) + i\psi(x,y)$.	5
	b)	Check the condition for orthogonality of the family of curves $u(x,y) = C_1$ and $v(x,y) = C_2$, when $f(z) = u + iv = (x^4 - 6x^2y^2 + y^4) + i(4x^3y - 4xy^3)$, where C_1, C_2 are real constants.	5
3.		Test the analyticity of $f(z) = \frac{x^3+xy^2+x}{x^2+y^2} + i\frac{x^2y+y^3-y}{x^2+y^2}$.	5
4.		Find the linear fractional transformation that maps the points $-1, 0, 1$ on the z-plane onto the points $-1, -i, 1$, respectively, on the w-plane. Also, find the image of the unit circle $\lvert z\rvert = 1$ under this transformation.	10
5.		Find the image of the rectangular region $-1 \le x \le 2, -\pi < y < \pi$ under the following transformations: (i) $w = e^z$ and (ii) $w = \frac{1}{z}$. Also, sketch the regions.	10
6.		Determine the points where $e^{z^5 - 80z}$ is not conformal.	5

*****